Regional gravity anomaly separation using wavelet transform and spectrum analysis

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Abstract
In this paper, we propose the separation of the regional gravity anomaly using wavelet multi-scale analysis and choose rational decomposition results based on the spectrum analysis and their depth estimation results. The isotropic, symmetrical and low-pass wavelet filter, Halo wavelet, is used for wavelet transform and regional anomaly separation. The anomaly separation method is verified by the synthetic model of combined cuboids. It has also been used for regional gravity separation and deep structure study in the Dadang area, North China. The regional anomalies are obtained from large-scale wavelet decomposition results. The average depths estimated from the radial power spectrum and the stratum density are used to establish the relationship between the separation anomaly and buried geologic units. The results from seismic data and magnetotellurics inversion are used to assist the regional anomaly interpretation. From the wavelet analysis and spectrum comparisons, the regional anomalies caused by Moho and Cenozoic basement are given for further deep structure analysis.

Keywords: regional anomaly separation, wavelet transform, spectrum analysis, depth estimation, Dadang area

1. Introduction

The observed gravity anomalies are the superposition of anomalies induced by geologic bodies at different depths. Regional residual anomaly separation is one of the important tasks in gravity inversion and interpretation.

A number of anomaly separation methods were developed based on different characteristics of regional and residual fields, such as the graphic smoothing and N-point smoothing (Wang et al 1991), polynomial surface fitting (Beltrao et al 1991), minimum curvature method (Mickus et al 1991), finite element analysis (Mallick and Sharma 1999), the stripping method (Weiland 1989). Li and Oldenburg (1998) proposed to separate the regional anomaly using a 3D magnetic inversion algorithm. Based on different spectral characteristics of gravity and magnetic anomalies, filters can be designed for anomaly separation, e.g. the Wiener filtering (Pawlowski and Hansen 1990), wavelength filtering (Kane 1985), band pass filtering (Ridsdill-Smith 1998) and preferential continuation filtering (Pawlowski 1995).

In recent years, the wavelet transform has widely been used in gravity data processing and interpretation due to the good property of multi-scale analysis, and it has become an important method of the anomaly separation. Fedi and Quarta (1998) used a discrete wavelet transform to separate the regional potential fields, and determined the rational decomposition results as a regional anomaly by minimum entropy compactness criterion. Ucan et al (2000) also used the multi-scale wavelet transform to separate the regional anomaly field and achieved satisfactory results in the synthetic model test. Yang et al (2001) analysed the gravity data of China using the discrete wavelet transform and interpreted...
the geological implications of the decomposition results. The multi-scale wavelet analysis can also be used in data denoising (Lyrio et al. 2004), geological boundary locating (Martelet et al. 2001) and source parameter inversion (Sailhac and Gibert 2003).

Besides the Euclidean wavelets, the spherical wavelets method has been developed in recent ten years (Freeden and Windheuser 1996, 1997). The spherical wavelet transform has similar multi-scale analysis property as the Euclidean wavelet transform, and it can be expressed by the convolution of a signal with a dilation and rotation of a spherical mother wavelet on a sphere. Compared with the Euclidean wavelets, spherical wavelets are widely used in large-scale data analysis, especially for the spherical earth. It has been used to study the global gravity field (Fengler et al. 2004), earth magnetic field (Freeden et al. 1998) and earth inner structure (mass density distribution) (Michel 2005).

The traditional spectrum analysis is usually used to assist wavelet analysis and interpretation of gravity and magnetic anomalies. Albora and Ucan (2001) present a synthetic example of gravity anomaly separation using wavelets, and estimate the average depth of buried bodies from the spectrum. Qiu et al (2007) discuss the ability of the wavelet transform to improve the resolution of gravity anomaly and use depth estimation from spectrum analysis to analyse the wavelet decomposition results. It has been proven that the depth estimation results from spectrum analysis are powerful assistance to wavelet multi-scale analysis. In this paper, the theory of wavelet transform and spectrum analysis was summarized and applied to the synthetic data and real data of Dagang area for gravity anomaly separation.

2. Theory of wavelet transform and spectrum analysis

2.1. Wavelet transform

Assuming that \( f(x) \) is a square integrable function, its wavelet transform can be expressed as

\[
WT_f(s, b) = \frac{1}{\sqrt{s}} \int_R f(x) \psi\left(\frac{b-x}{s}\right) \, dx = f(x) * \psi_s(x),
\]

where \( \psi_s(x) \) is the wavelet basis or the mother wavelet function, \( s > 0 \) is the scale factor, \( b \) is the translation parameter, \( R \) is the integration domain, \( \psi_s(x) \) is the dilation of wavelet basis and \( \psi_s(x) = \frac{1}{\sqrt{s}} \psi\left(\frac{x}{s}\right) \). (\( * \) means convolution).

In the frequency domain, equation (1) can be equivalently expressed as

\[
WT_f(s, b) = \sqrt{\frac{s}{2\pi}} \int_R F(\vec{k}) \Psi(s\vec{k}) e^{i\vec{k}\cdot\vec{b}} \, d\vec{k},
\]

where \( \Psi(\vec{w}) \) is the Fourier transform of \( \psi(x) \), \( \sqrt{s} \Psi(\vec{k}) \) is the Fourier transform of \( \psi_s(x) \).

Generally, the scale factor can be connected with the frequency by

\[
F_s = \frac{F_c}{s \cdot \Delta},
\]

where \( F_s \) is the equivalent frequency of wavelet transform at scale \( s \), \( F_c \) is the centre frequency of the wavelet basis function, and \( \Delta \) is the sampling rate.

From the frequency domain expression (equation (2)), the wavelet transform of the signal \( f(x) \) can be viewed as the filtering result with the wavelet filter at different scales (Yang 1999) or filter banks operation (Strang and Nguyen 1997). Generally, a large-scale wavelet transform can be used to separate the regional anomaly. Wavelets can be selected for anomaly analysis according to some criteria, such as similarity between signal and mother wavelets (Xu et al. 2004), minimum entropy compactness criterion (Fedi and Quarta 1998). In this paper, we select the wavelet according to its frequency response character. Based on the knowledge of the spectral character of anomalies, a low-pass and isotropic wavelet filter is more appropriate for the regional anomaly separation. Here, we studied the properties of the Halo wavelet in the frequency domain and applied it to separate the regional anomaly.

The Halo wavelet basis function is a modification of the Morlet wavelet (Kirby 2005). It can be expressed in the frequency domain as

\[
\Psi(\vec{k}) = e^{-|\vec{k}|(\vec{k}_0)^3/2}.
\]

Its spectrum character is shown in figure 1. The Halo wavelet basis is symmetrical and isotropic in the frequency domain. It is a low-pass wavelet filter with a small \( k_0 \). According to uncertainty, the bandwidth and the centre frequency of the dilated wavelet decrease when the scale increases. Therefore it is necessary to select the wavelet transform at a proper scale in order to get low-frequency regional anomalies.

From the definition of the wavelet transform, it can be computed by convolution in the space domain or multiplication in the frequency domain. We compute the wavelet transform in the frequency domain based on equation (2), and the implementation steps are listed below:

1. Compute the Fourier transform \( G(\vec{k}) \) of the original anomaly signal \( g(\vec{x}) \).
2. Multiply the anomaly spectrum \( G(\vec{k}) \) with Halo wavelet \( \Psi(\vec{k}) \) in the frequency domain, and get the wavelet transform at scale \( s = 1 \): \( W(\vec{k}) = G(\vec{k}) \times \Psi(\vec{k}) \).
3. Compute the inverse Fourier transform of \( W(\vec{k}) \) and get the wavelet transform result \( u(\vec{x}) \) in the space domain.
4. Calculate the wavelet transform of different scales with the dilated wavelet basis, and get the result of the wavelet transform result at different scales following steps (2) and (3).

The maximum decomposition scale relates the dimension of the original data, and the scale can take continuous values with a maximum of half of the data dimension. Here we take \( s = 2^n \) in the wavelet decomposition \( (n = 0, 0.5, 1, 1.5, \ldots, \) the order of decomposition).
2.2. Spectrum analysis and depth estimation

Spector and Grant (1970) studied the relationship between the energy spectrum of anomalies and the average depth of source bodies under a statistic assumption. It provided a foundation for anomaly source parameter estimation and filter designation for anomaly separation (Dolmaz et al. 2005, Wang et al. 1991).

The energy spectrum of anomalies can be presented by the formula:

$$\langle E(k) \rangle = 4 \pi M^2 \langle e^{-2hk} \rangle \langle 1 - e^{-tk} \rangle \langle S^2(k) \rangle,$$

where $\langle \rangle$ stands for ensemble average, $M$ is the magnetic moment/unit volume, $h$ is the depth to the top of source body, $t$ is the thickness of the source body, $k$ is the radial wave number, $S(k)$ is the factor for the horizontal size of the source body.

It was found that the depth factor $\langle e^{-2hk} \rangle$ dominates the spectrum, the effect of the extension factor $\langle 1 - e^{-tk} \rangle$ and the horizontal factor $\langle S^2(k) \rangle$ is comparatively small, especially in low-frequency bands. The energy spectrum can be simplified as

$$E(r) \approx Ae^{-2kh},$$

and

$$\ln(E(r)) \approx -2hk + A',$$

where $A$ and $A'$ are constant coefficients, $\bar{h}$ is the average depth of the source body.

In practice, the linear fitting results of different spectrum segments are plotted on the semi-log plot of energy spectrum versus radial wave number for convenience. The slopes of the best-fit straight lines of spectrum segments of logarithm energy spectrum versus radial wave number plot indicate the average depth of the sources.

3. Synthetic model experiment

We designed a cuboids combination model for the gravity field separation experiment by the wavelet transform. This model consists of six cuboids: the largest one is located in the deepest part to simulate the regional anomaly, and the other five smaller ones with the same size are located in the shallower part at the centre and four corners of the survey to simulate the local anomaly field (figure 2(a)). The relevant parameters are listed in table 1. The coordinate origin is located at the centre of the survey, the grid spacing is 0.1 km, and the survey area is 100 km × 100 km. Using the forward calculation formula of rectangular bodies (Blakely 1995), we calculated the gravity anomalies of the model and the corresponding regional and local anomalies, which are respectively shown in figures 2(b)–(d).

From the spectral analysis of the total, regional and local anomalies (figure 3), the anomaly energy is mainly concentrated in the low-frequency band (0–0.4 rad km$^{-1}$). The regional anomaly energy is dominated in the low-frequency band (0–0.4 rad km$^{-1}$), while the local anomaly energy is dominated in the mid-high frequency band (above 0.4 rad km$^{-1}$). The two anomalies have different spectral distribution characteristics. Therefore, it is feasible to separate anomalies of different frequency bands.

The spectrum of the total gravity anomaly can be divided into three segments in the following frequency ranges: 0–0.05, 0.05–0.60 and above 0.60 rad km$^{-1}$. They represent the regional anomaly with low frequency and high energy, the local anomaly with intermediate and high frequencies, and the high frequency signal characterized with very small energy, respectively. We choose the Halo wavelet basis to process the gravity anomaly based on the spectral character. Taking $k_0 = 0.6$ and the corresponding scale $s = 2^{5.5}$, the transform result is...
Figure 2. (a) Synthetic model. (b) Total gravity anomaly induced by the model. (c) Regional gravity anomaly induced by the largest and deepest cuboids. (d) Local gravity anomaly induced by the five smaller cuboids.

Table 1. Model parameters.

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taken as the regional anomaly, and the difference between this result and the original anomaly is taken as the local anomaly (figure 4). It has achieved satisfied separation results compared with the theoretical anomalies (figures 2(c) and (d)).
4. Real data example

Dagang area is located in the mid-north part of the Bohaiwan Basin, North China (figure 5(a)). It is an important petroleum production area in China, and it consists of the Cangdong Uplift, Huanghua Depression, Chengning Uplift and Shaleitian Uplift (figure 5(b)).

In recent years, the gravity and magnetic method has played an important role in the pre-Cenozoic structure study and oil exploration of residual basins (Liu 2001). Based on the analysis of rock physics properties, the separated regional anomalies induced by the deep geology structures were used in pre-Cenozoic structure inversion (Xu et al 2007).

The Bouguer gravity anomaly and tectonic scheme map of the area (figure 5(b)) show that the overall distribution of the gravity anomaly is consistent with the uplift and depression patterns. The Huanghua Depression is a low-value gravity anomaly area, and it is surrounded by three high-value anomalies which correspond to the three uplifts. The local gravity undulation within the Cenozoic sedimentary basin (Huanghua Depression) corresponds to smaller sags and salients, forming a superimposed multi-scale gravity anomaly pattern.

According to the statistic data of rock and stratum densities in Dagang area (Hao et al 2008), there are three main density contrast interfaces: (1) the interface between Paleogene and Neogene with a density contrast about 200–300 kg m$^{-3}$; (2) the interface between Paleogene-Cretaceous and Jurassic with a density contrast about 150 kg m$^{-3}$ and (3) the Moho discontinuities with a density contrast about 400 kg m$^{-3}$. The gravity anomalies in Dagang area are the superposition of anomalies induced mainly by the above three density contrast interfaces. In order to study the pre-Cenozoic structures, we need to extract the gravity anomaly induced by the density contrast interface between Paleogene-Cretaceous and Jurassic. We use the wavelet transform and spectrum analysis to separate the Bouguer gravity anomaly based on the density structure, and choose the rational anomaly separation result.

Figure 6 displays the spectrum of the gravity anomaly. The energy of the anomaly is mainly concentrated in the low-frequency band. The spectrum can be divided into three segments: frequency bands of 0–0.1, 0.1–0.32 and 0.32–0.8 rad km$^{-1}$. The average depths of burial geology bodies estimated from the three spectrum segments are 29.3 km, 9.7 km and 2.9 km, respectively. The estimated average depths were coincident with the results from other geophysical data. The depth of Neogene revealed by seismic data interpretation is about 1.5–3 km (Deng 1996; figure 7), and...
the Moho depth is about 30–34 km (Zhou et al. 2003, Wang et al. 2002). The magnetotelluric (MT) investigation results (figure 7) indicate that the depth of the deep structure (Mesozoic-Paleozoic) is about 7–11 km in Qikou Sag and less than 3 km in Chengning and Cangdong Uplifts. So the spectrum of ultra-low and mid-low frequency bands corresponds to the anomaly induced by Moho and pre-Cenozoic structures, and the other high-frequency bands mainly reflect the anomaly induced by shallow structures and noises. The regional anomaly induced by a deep structure must be separated for the pre-Cenozoic structure study (Hao et al. 2008).

We use the Halo wavelet basis to analyse and separate the gravity anomaly in Dagang area based on the anomaly spectrum characteristics and the depth estimation results. The centre frequency of the Halo wavelet is $k_0 = 0.8$, the scale factor is $2^5$, $2^6$ and $2^7$, respectively. The decomposition result is shown in figures 8(a)–(c), and the corresponding spectrum in figure 6. Compared with the original anomaly spectrum, the energy of the large-scale wavelet decomposition is mainly concentrated in the low-frequency part compared with the original anomaly spectrum. We estimate the average depth from the spectrum of the wavelet decomposition results. The

**Figure 5.** (a) The location of the Dagang area. (b) Gravity anomaly and tectonic scheme map in Dagang area, and the red cross indicates the magnetotelluric site.

**Figure 6.** Spectrum of gravity in Dagang area (black line) and the wavelet decomposition results (red, blue and purple lines correspond to the wavelet decomposition results at scales $2^5$, $2^6$, $2^7$, respectively). (a) Depth estimation results of the gravity anomaly spectrum in Dagang area. (b), (c) and (d) Depth estimation results of wavelet decomposition at scales $2^5$, $2^6$, $2^7$. 284
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Figure 7. Magnetotelluric inversion profile in Dagang. The position of the profile is shown in figure 5(b), the green line is the bottom of Neogene from seismic data, and the broken blue line is the bottom of Mesozoic-Paleozoic interpreted from MT inversion.

Figure 8. Wavelet decomposition results of the gravity anomaly in Dagang area at different scales: (a) scale: $s = 2^5$; (b) scale: $s = 2^6$ and (c) scale: $s = 2^7$.

depth estimation results of the fifth-order wavelet transform are 29.4 km and 10.2 km, which reflect the anomaly signals from middle-deep geologic bodies of pre-Cenozoic and Moho discontinuity. The estimated depth of the sixth-order wavelet transform is 27.1 km, which corresponds to deep anomalies near the Moho discontinuity. The estimated depth of the seventh-order result is 43.2 km, which reflects the anomaly character from deep geologic bodies in the upper mantle and represents the overall trend of gravity distribution in the study region.

The difference between the fifth- and sixth-order wavelet analysis results (figure 9) eliminated the anomaly trend induced by deep anomaly bodies. Compared with the origin gravity anomaly and tectonic map, the anomalies of Cenozoic sedimentary basin (sags and salients) are effectively suppressed. The spectrum (figure 10) also indicates that the energy of the extracted anomaly is mainly concentrated in the middle-low frequency band. It has the same character with the original anomaly in the middle-low frequency band and reflects the deep geological structures. The depth estimation of the spectrum is the same as the second segments of the original anomaly. It represents the anomaly induced by the second density interface (pre-Cenozoic geologic bodies).

In summary, the wavelet transform results of different scales provide the anomaly information of geologic bodies at different depths. The depth estimation results from the separated anomaly spectrum can show the corresponding geology implications. We can choose a proper regional anomaly according to the research purpose by comparing different wavelet transform results and their spectrum characters. In Dagang area, the fifth-order wavelet transform result mainly reflects the anomaly of the mid-lower crust geologic structure; the sixth-order result mainly reflects the anomaly induced by the density contrast near the
5. Conclusions

There is no simple answer to anomaly separation (Nabighian et al. 2005). In this paper, we propose to separate the regional anomaly using the wavelet transform according to the spectrum analysis and their depth estimation results. The isotropic and low-pass wavelet filter, Halo wavelet, is used in the synthetic and real data processing. The separation test on the synthetic model indicates that the wavelet analysis can separate the anomaly effectively.

The spectrum of the gravity anomaly of Dagang area can be divided into three segments, corresponding to the anomalies induced by three major density interface structures in the area. The fifth-, sixth- and seventh-order wavelet transform results show different regional anomalies of buried geology structures. The fifth-order result mainly reflects the anomaly induced by the deep geologic bodies including pre-Cenozoic and Moho. The results from the sixth- and seventh-order analyses represent the gravity anomalies induced by the density contrast across the Moho and the geologic bodies in the upper mantle, respectively. The difference between the results of the fifth- and sixth-order wavelet transform represents the anomaly induced by Pre-Cenozoic structures in this area. It can be used in further study of the pre-Cenozoic structure.

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